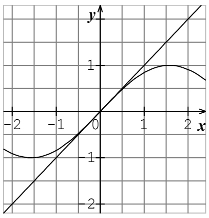
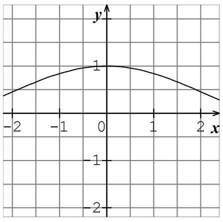
Limites à connaître

Taux de variation



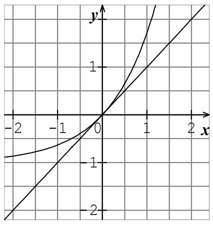


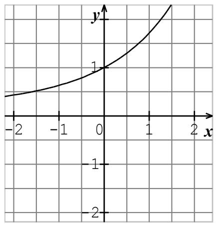
$$\boxed{\lim_{x \to 0} \frac{\sin(x)}{x} = 1}$$

Démonstration :

$\forall\, x \in \mathbb{R}^*,\ \dfrac{\sin(x)}{x} = \dfrac{\sin(x) - \sin(0)}{x - 0}$

et $\displaystyle\lim_{x \to 0} \frac{\sin(x) - \sin(0)}{x - 0} = \sin'(0) = \ldots$



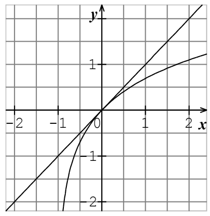


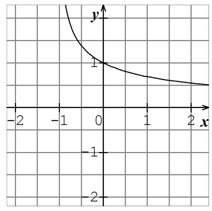
$$\boxed{\lim_{x \to 0} \frac{e^x - 1}{x} = 1}$$

Démonstration :

$\forall\, x \in \mathbb{R}^*,\ \dfrac{e^x - 1}{x} = \ldots$

et $\displaystyle\lim_{x \to 0} \frac{\cdots\cdots\cdots}{\cdots\cdots\cdots} =$





$$\boxed{\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1}$$

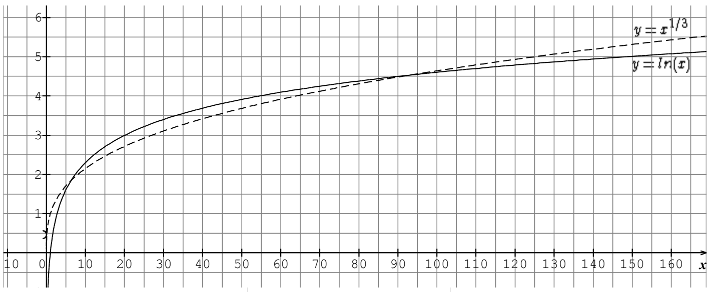
Démonstration :

$\forall\, x \in \mathbb{R}^*,\ \dfrac{\ln(x+1)}{x} = \ldots$

et $\displaystyle\lim_{x \to 0} \frac{\cdots\cdots\cdots}{\cdots\cdots\cdots} =$

Croissances comparées

Soit α un réel strictement positif :



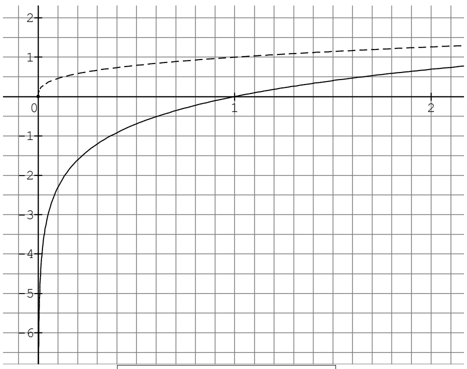
$$\boxed{\lim_{x \to +\infty} \frac{\ln(x)}{x^\alpha} = 0^+}$$

Démonstration : en étudiant la fonction *f* définie sur $]0\,;+\infty[$ par : $f(x) = \sqrt{x} - \ln(x)$...

...pour tout réel $x \geqslant 1$, $0 \leqslant \dfrac{\ln(x)}{x} \leqslant \dfrac{1}{\sqrt{x}}$

...on en déduit que $\displaystyle\lim_{x \to +\infty} \frac{\ln(x)}{x} = 0$

et comme pour tout $x > 0$, $\dfrac{\ln(x)}{x^\alpha} = \dfrac{\left(\dfrac{1}{\alpha}\right)\ln\left(x^\alpha\right)}{x^\alpha}$...



$$\boxed{\lim_{x \to 0^+} x^\alpha \ln(x) = 0^-}$$

Démonstration : pour tout réel $x > 0$, $x^\alpha \ln(x) = \dfrac{-\ln\left(\dfrac{1}{x}\right)}{\left(\dfrac{1}{x}\right)^\alpha}$

en posant $X = \dfrac{1}{x}$...

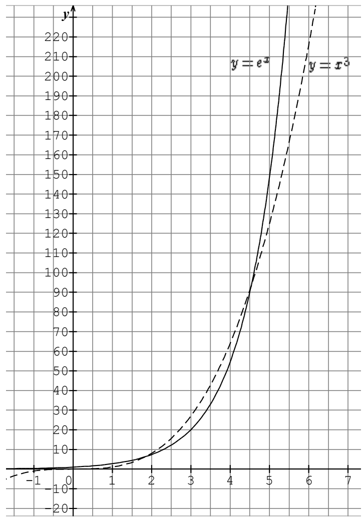
$$\boxed{\lim_{x \to +\infty} \frac{e^x}{x^\alpha} = +\infty}$$

Démonstration :

pour tout réel $x > 0$,

$\dfrac{e^x}{x^\alpha} = e^{x - \alpha\ln(x)} = e^{x\left(1 - \frac{\alpha\ln(x)}{x}\right)}$

...



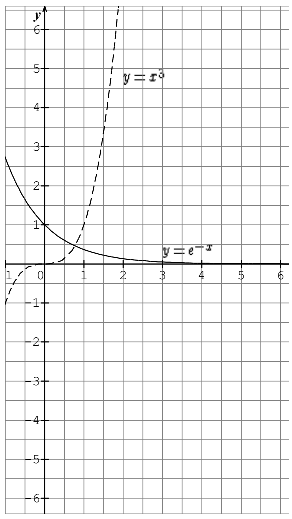
$$\boxed{\lim_{x \to +\infty} x^\alpha e^{-x} = 0}$$

Démonstration :

pour tout réel $x > 0$,

$x^\alpha e^{-x} = \dfrac{x^\alpha}{e^x} = \dfrac{1}{\dfrac{e^x}{x^\alpha}}$

...



On peut retenir :